INTEGRATION BEE FINALS

The finals are conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

Contestants must circle their final answer. Contestants do **not** need to include the constant of integration $+C$ in their answer.

The last person remaining wins an additional $75 and will be crowned the **Integration Champion**!

INTEGRAL #1

READY,

GET SET,...

2:30

INTEGRAL #1

$$\int \frac{x^2 - x^6}{1 - x^2} \, \mathrm{d}x$$

2:30

INTEGRAL #1

$$\int \frac{x^2 - x^6}{1 - x^2}\,dx$$

$$= \int \frac{x^2(1 - x^2)(1 + x^2)}{1 - x^2}\,dx$$

$$= \int (x^2 + x^4)\,dx$$

$$= \boxed{\frac{x^3}{3} + \frac{x^5}{5} + C}$$

INTEGRAL #2

READY,

GET SET,...

2:30

INTEGRAL #2

$$\int \frac{1+\frac{x}{x+1}}{x}\,dx$$

2:30

INTEGRAL #2

$$\int \frac{1+\frac{x}{x+1}}{x}\,dx$$

$$= \int \frac{1+\frac{x}{x+1}}{x} \cdot \frac{x+1}{x+1}\,dx$$

$$= \int \frac{2x+1}{x^2+x}\,dx$$

$$= \int \frac{1}{u}\,du \qquad u = x^2+x, \quad du = (2x+1)\,dx$$

$$= \boxed{\ln|x^2+x| + C}$$

INTEGRAL #3

READY,

GET SET,...

2:30

INTEGRAL #3

$$\int \frac{x}{x^4+2x^2+1}\,\mathrm{d}x$$

2:30

INTEGRAL #3

$$\int \frac{x}{x^4+2x^2+1}\,dx$$

$$= \int \frac{x}{(x^2+1)^2}\,dx$$

$$= \frac{1}{2}\int \frac{1}{u^2}\,du \qquad u = x^2+1, \quad du = 2x\,dx$$

$$= -\frac{1}{2u} + C$$

$$= \boxed{-\frac{1}{2(x^2+1)} + C}$$

INTEGRAL #4

READY,

GET SET,...

2:30

INTEGRAL #4

$$\int \frac{x^2+x-1}{(2x+1)^2-5}\,\mathrm{d}x$$

2:30

INTEGRAL #4

$$\int \frac{x^2+x-1}{(2x+1)^2-5}\,dx$$

$$= \int \frac{x^2+x-1}{4x^2+4x-4}\,dx$$

$$= \int \frac{1}{4}\,dx$$

$$= \boxed{\frac{1}{4}x + C}$$

INTEGRAL #5

READY,

GET SET,...

2:30

INTEGRAL #5

$$\int \frac{1}{\sqrt{x+7}+\sqrt{x}}\,dx$$

2:30

INTEGRAL #5

$$\int \frac{1}{\sqrt{x+7}+\sqrt{x}}\,dx$$

$$= \int \frac{1}{\sqrt{x+7}+\sqrt{x}} \cdot \frac{\sqrt{x+7}-\sqrt{x}}{\sqrt{x+7}-\sqrt{x}}\,dx$$

$$= \int \frac{\sqrt{x+7}-\sqrt{x}}{7}\,dx$$

$$= \boxed{\frac{2}{21}\left((x+7)^{3/2}-x^{3/2}\right)+C}$$

INTEGRAL #6

READY,

GET SET,...

2:30

INTEGRAL #6

$$\int \sqrt{4+\left(x^2-\frac{1}{x^2}\right)^2}\,\mathrm{d}x$$

2:30

INTEGRAL #6

$$\int \sqrt{4+\left(x^2-\frac{1}{x^2}\right)^2}\,dx$$

$$=\int \sqrt{4+x^4-2+\frac{1}{x^4}}\,dx$$

$$=\int \sqrt{\left(x^2+\frac{1}{x^2}\right)^2}\,dx$$

$$=\int \left(x^2+\frac{1}{x^2}\right)dx = \boxed{\frac{x^3}{3}-\frac{1}{x}+C}$$

INTEGRAL #7

READY,

GET SET,...

2:30

INTEGRAL #7

$$\int \frac{e^{(2x-1)/x}}{x^2}\,dx$$

2:30

INTEGRAL #7

$$\int \frac{e^{(2x-1)/x}}{x^2}\,dx$$

$$= \int \frac{e^{2-1/x}}{x^2}\,dx$$

$$= \int e^u\,du \qquad u = 2 - 1/x,\ du = \frac{1}{x^2}\,dx$$

$$= e^u + C$$

$$= \boxed{e^{2-1/x} + C}$$

INTEGRAL #8

READY,

GET SET,...

2:30

INTEGRAL #8

$$\int \frac{e^x}{e^x + 1} \, dx$$

2:30

INTEGRAL #8

$$\int \frac{e^x}{e^x + 1}\,dx$$

$$= \int \frac{1}{u}\,du \qquad u = e^x + 1, \quad du = e^x\,dx$$

$$= \ln|u| + C$$

$$= \boxed{\ln(e^x + 1) + C}$$

INTEGRAL #9

READY,

GET SET,...

2:30

INTEGRAL #9

$$\int \tan x \ln \cos x \, dx$$

2:30

INTEGRAL #9

$$\int \tan x \ln\cos x \, dx$$

$$= -\int u \, du \qquad u = \ln\cos x, \quad du = \frac{1}{\cos x}(-\sin x)\,dx = -\tan x \, dx$$

$$= -\frac{u^2}{2} + C$$

$$= \boxed{-\frac{1}{2}(\ln\cos x)^2 + C}$$

INTEGRAL #10

READY,

GET SET,...

2:30

INTEGRAL #10

$$\int \tan x \sec^4 x \sqrt{1 + \sec^4 x}\, dx$$

2:30

INTEGRAL #10

$$\int \tan x \sec^4 x \sqrt{1+\sec^4 x}\, dx$$

$$= \frac{1}{4}\int \sqrt{u}\, du \qquad u = 1+\sec^4 x, \quad du = 4\sec^3 x(\sec x \tan x)\, dx$$

$$= \frac{1}{6}u^{3/2} + C$$

$$= \boxed{\frac{1}{6}\left(1+\sec^4 x\right)^{3/2} + C}$$

INTEGRAL #11

READY,

GET SET,...

2:30

INTEGRAL #11

$$\int \cos^5 x \sin 2x \, \mathrm{d}x$$

2:30

INTEGRAL #11

$$\int \cos^5 x \sin 2x \, dx$$

$$= \int \cos^5 x (2 \sin x \cos x) \, dx$$

$$= 2 \int \cos^6 x \sin x \, dx \qquad u = \cos x, \quad du = -\sin x \, dx$$

$$= -2 \int u^6 \, du$$

$$= -\frac{2}{7} u^7 + C = \boxed{-\frac{2}{7} \cos^7 x + C}$$

INTEGRAL #12

READY,

GET SET,...

2:30

INTEGRAL #12

$$\int \frac{2x^2+7x+4}{x+3}\,\mathrm{d}x$$

2:30

INTEGRAL #12

$$\int \frac{2x^2+7x+4}{x+3}\,dx$$

$$= \int \left(2x+1+\frac{1}{x+3}\right)dx \qquad \text{Long Division}$$

$$= \boxed{x^2+x+\ln|x+3|+C}$$

INTEGRAL #13

READY,
GET SET,...

2:30

INTEGRAL #13

$$\int \frac{\sin 2x}{(1+\cos 4x)^2}\,dx$$

2:30

$$\int \frac{\sin 2x}{(1+\cos 4x)^2}\,dx$$

$$= \int \frac{\sin 2x}{(2\cos^2 2x)^2}\,dx$$

$$= \frac{1}{4}\int \frac{\sin 2x}{\cos^4 2x}\,dx$$

$$= -\frac{1}{8}\int \frac{1}{u^4}\,du \qquad u = \cos 2x, \quad du = -2\sin 2x\,dx$$

$$= \frac{1}{24u^3} + C = \boxed{\frac{1}{24\cos^3 2x} + C}$$

INTEGRAL #14

READY,

GET SET,...

2:30

INTEGRAL #14

$$\int x \csc^2 x \, dx$$

2:30

INTEGRAL #14

$$\int x \csc^2 x \, dx$$

$$= -x \cot x + \int \cot x \, dx$$

Integration by parts:

$$u = x \qquad dv = \csc^2 x \, du$$
$$du = dx \qquad v = -\cot x$$

$$= \boxed{-x \cot x + \ln|\sin x| + C}$$

INTEGRAL #15

READY,

GET SET,...

2:30

INTEGRAL #15

$$\int \frac{x}{x^3+3x^2+3x+1}\,\mathrm{d}x$$

2:30

INTEGRAL #15

$$\int \frac{x}{x^3+3x^2+3x+1}\,dx$$

$$= \int \frac{x}{(x+1)^3}\,dx$$

$$= \int \frac{u-1}{u^3}\,du \qquad u = x+1, \quad du = dx, \quad u-1 = x$$

$$= \int \left(u^{-2}-u^{-3}\right)du$$

$$= -\frac{1}{u}+\frac{1}{2u^2}+C = \boxed{-\frac{1}{x+1}+\frac{1}{2(x+1)^2}+C}$$