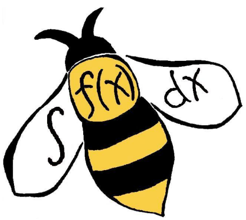
HIGH SCHOOL ROUND ONE



You will have **two minutes** to evaluate each of the fifteen definite integrals that will be displayed one at a time on this screen. **All answers must be simplified.** At the end of the two minutes, all hands must go up and judges will grade your answers immediately. For each correct answer, you will receive one raffle ticket to be entered for prizes that will be drawn after dinner.

At most five participants will move to the finals – to be determined by the total number of correct answers and tiebreaking criteria if necessary. **Everyone moving to the finals will receive $25.**

INTEGRAL #1

READY,

GET SET,...

2:00

INTEGRAL #1

$$\int_0^1 \left((x+1)^2+(x+2)^2+(x+3)^2\right)\,dx$$

INTEGRAL #1

$$\int_0^1 \left((x+1)^2+(x+2)^2+(x+3)^2\right)\,dx$$

$$= \int_0^1 (3x^2+12x+14)\,dx$$

$$= \left[x^3+6x^2+14x\right]_0^1$$

$$= \boxed{21}$$

INTEGRAL #2

READY,

GET SET,...

2:00

INTEGRAL #2

$$\int_0^{\pi/3} \frac{\sin x}{(1+2\cos x)^2}\,dx$$

INTEGRAL #2

$$\int_0^{\pi/3} \frac{\sin x}{(1+2\cos x)^2}\,dx$$

$$= -\frac{1}{2}\int_3^2 \frac{1}{u^2}\,du \qquad u = 1 + 2\cos x,\ \ du = -2\sin x\,dx$$

$$= \frac{1}{2}\left[\frac{1}{u}\right]_3^2$$

$$= \boxed{\frac{1}{12}}$$

INTEGRAL #3

READY,

GET SET,...

2:00

INTEGRAL #3

$$\int_1^4 \frac{\sqrt{x}+4x^2}{x}\,dx$$

INTEGRAL #3

$$\int_1^4 \frac{\sqrt{x} + 4x^2}{x}\,dx$$

$$= \int_1^4 \left(x^{-1/2} + 4x\right)\,dx$$

$$= \Big[2x^{1/2} + 2x^2\Big]_1^4$$

$$= \boxed{32}$$

INTEGRAL #4

READY,
GET SET,...

2:00

INTEGRAL #4

$$\int_{\sqrt{\pi}}^{\sqrt{4\pi/3}} x \sec(x^2 - \pi) \tan(x^2 - \pi)\, dx$$

INTEGRAL #4

$$\int_{\sqrt{\pi}}^{\sqrt{4\pi/3}} x\sec(x^2-\pi)\tan(x^2-\pi)\,dx$$

$$= \frac{1}{2}\int_0^{\pi/3} \sec u \tan u\,du \qquad u = x^2-\pi,\ \ du = 2x\,dx$$

$$= \frac{1}{2}\Big[\sec u\Big]_0^{\pi/3}$$

$$= \boxed{\frac{1}{2}}$$

INTEGRAL #5

READY,

GET SET,...

2:00

INTEGRAL #5

$$\int_0^{\ln 8} \frac{2}{e^{x/3}}\,dx$$

INTEGRAL #5

$$\int_0^{\ln 8} \frac{2}{e^{x/3}}\,dx$$

$$= \int_0^{\ln 8} 2e^{-x/3}\,dx = \left[-6e^{-x/3}\right]_0^{\ln 8}$$

$$= -6\left(e^{\ln 8^{-1/3}} - 1\right)$$

$$= -6\left(\frac{1}{2} - 1\right) = \boxed{3}$$

INTEGRAL #6

READY,

GET SET,...

2:00

INTEGRAL #6

$$\int_{9}^{25} \frac{\left(5-\sqrt{x}\right)^{5}}{\sqrt{x}}\,\mathrm{d}x$$

INTEGRAL #6

$$\int_9^{25} \frac{(5-\sqrt{x})^5}{\sqrt{x}}\,dx$$

$$= -2\int_2^0 u^5\,du \qquad u = 5-\sqrt{x},\;\; du = -\frac{1}{2\sqrt{x}}\,dx$$

$$= -2\left[\frac{u^6}{6}\right]_2^0$$

$$= \boxed{\frac{64}{3}}$$

INTEGRAL #7

READY,

GET SET,...

2:00

INTEGRAL #7

$$\int_{-3}^{6} \sqrt[3]{3x+9}\,dx$$

INTEGRAL #7

$$\int_{-3}^{6} \sqrt[3]{3x+9}\,dx$$

$$= \frac{1}{3}\int_{0}^{27} u^{1/3}\,du \qquad u = 3x+9,\ \ du = 3\,dx$$

$$= \left[\frac{u^{4/3}}{4}\right]_{0}^{27}$$

$$= \boxed{\frac{81}{4}}$$

INTEGRAL #8

READY,
GET SET,...

2:00

INTEGRAL #8

$$\int_0^{\pi} (x^3 + \sin 3x)(x^2 + \cos 3x)\, dx$$

INTEGRAL #8

$$\int_0^{\pi} (x^3 + \sin 3x)(x^2 + \cos 3x)\, dx$$

$$= \frac{1}{3}\int_0^{\pi^3} u\, du \quad u = x^3 + \sin 3x, \;\; du = (3x^2 + 3\cos 3x)\, dx$$

$$= \left[\frac{u^2}{6}\right]_0^{\pi^3}$$

$$= \boxed{\frac{\pi^6}{6}}$$

INTEGRAL #9

READY,

GET SET,...

2:00

INTEGRAL #9

$$\int_{-1}^{0} 3x^2(x^3+1)^{2016}\,dx$$

INTEGRAL #9

$$\int_{-1}^{0} 3x^2(x^3+1)^{2016}\,dx$$

$$= \int_0^1 u^{2016} \qquad u = x^3+1,\ du = 3x^2\,dx$$

$$= \left[\frac{u^{2017}}{2017}\right]_0^1$$

$$= \boxed{\frac{1}{2017}}$$

INTEGRAL #10

READY,

GET SET,...

2:00

INTEGRAL #10

$$\int_0^{\pi/4} \frac{\sin^3 x + \sec^3 x}{\sec x}\,dx$$

INTEGRAL #10

$$\int_0^{\pi/4} \frac{\sin^3 x + \sec^3 x}{\sec x}\, dx$$

$$= \int_0^{\pi/4} \left(\frac{\sin^3 x}{\sec x} + \frac{\sec^3 x}{\sec x} \right) dx$$

$$= \int_0^{\pi/4} \left(\sin^3 x \cos x + \sec^2 x \right) dx$$

$$= \left[\frac{\sin^4 x}{4} + \tan x \right]_0^{\pi/4} = \boxed{\frac{17}{16}}$$

INTEGRAL #11

READY,

GET SET,...

2:00

INTEGRAL #11

$$\int_0^1 \sqrt{x\sqrt{x\sqrt{x}}}\,dx$$

INTEGRAL #11

$$\int_0^1 \sqrt{x\sqrt{x\sqrt{x}}}\,dx$$

$$= \int_0^1 \left(x \cdot \left(x \cdot x^{1/2}\right)^{1/2}\right)^{1/2} dx$$

$$= \int_0^1 x^{7/8}\,dx$$

$$= \left[\frac{8x^{15/8}}{15}\right]_0^1 = \boxed{\frac{8}{15}}$$

INTEGRAL #12

READY,
GET SET,...

2:00

INTEGRAL #12

$$\int_0^{\pi/2} (x+1)\sin x \, dx$$

INTEGRAL #12

$$\int_0^{\pi/2} (x+1)\sin x\,dx$$

integrate by parts: $u = x+1 \quad dv = \sin x\,dx$

$du = dx \quad v = -\cos x$

$$= \Big[-(x+1)\cos x\Big]_0^{\pi/2} + \int_0^{\pi/2} \cos x\,dx$$

$$= 1 + \Big[\sin x\Big]_0^{\pi/2} = \boxed{2}$$

INTEGRAL #13

READY,

GET SET,...

2:00

INTEGRAL #13

$$\int_{-1}^{2} \frac{x^2}{\sqrt{x^3+17}}\,dx$$

INTEGRAL #13

$$\int_{-1}^{2} \frac{x^2}{\sqrt{x^3+17}}\,dx$$

$$= \frac{1}{3}\int_{16}^{25} u^{-1/2}\,du \qquad u = x^3 + 17,\ \ du = 3x^2\,dx$$

$$= \left[\frac{2}{3}u^{1/2}\right]_{16}^{25}$$

$$= \boxed{\frac{2}{3}}$$

INTEGRAL #14

READY,

GET SET,...

2:00

INTEGRAL #14

$$\int_0^{\pi/14} \frac{\sin^2 7x}{1+\cos 7x}\,dx$$

INTEGRAL #14

$$\int_0^{\pi/14} \frac{\sin^2 7x}{1+\cos 7x}\,dx$$

$$= \int_0^{\pi/14} \frac{1-\cos^2 7x}{1+\cos 7x}\,dx = \int_0^{\pi/14} \frac{(1+\cos 7x)(1-\cos 7x)}{1+\cos 7x}\,dx$$

$$= \int_0^{\pi/14} (1-\cos 7x)\,dx$$

$$= \left[x - \frac{\sin 7x}{7}\right]_0^{\pi/14} = \boxed{\frac{\pi}{14} - \frac{1}{7} = \frac{\pi-2}{14}}$$

INTEGRAL #15

READY,

GET SET,...

2:00

INTEGRAL #15

$$\int_0^1 (1-x)(1+x)(1+x^2)(1+x^4)\,dx$$

INTEGRAL #15

$$\int_0^1 (1-x)(1+x)(1+x^2)(1+x^4)\,dx$$

$$= \int_0^1 (1-x^2)(1+x^2)(1+x^4)\,dx$$

$$= \int_0^1 (1-x^4)(1+x^4)\,dx$$

$$= \int_0^1 (1-x^8)\,dx = \left[x - \frac{x^9}{9}\right]_0^1 = \boxed{\frac{8}{9}}$$