

COLLEGE ROUND ONE



You will have **two minutes** to evaluate each of the fifteen definite integrals that will be displayed one at a time on this screen. **All answers must be simplified.** At the end of the two minutes, all hands must go up and judges will grade your answers immediately. For each correct answer, you will receive one raffle ticket to be entered for prizes that will be drawn after dinner.

At most five participants will move to the finals – to be determined by the total number of correct answers and tiebreaking criteria if necessary. **Everyone moving to the finals will receive \$25.**

INTEGRAL #1

**READY,
GET SET,...**

2:00

INTEGRAL #1

$$\int_1^2 \left(\frac{x+2}{x} \right)^2 dx$$

INTEGRAL #1

$$\int_1^2 \left(\frac{x+2}{x} \right)^2 dx$$

$$= \int_1^2 \left(1 + \frac{2}{x} \right)^2 dx = \int_1^2 \left(1 + \frac{4}{x} + \frac{4}{x^2} \right) dx$$

$$= \left[x + 4 \ln x - \frac{4}{x} \right]_1^2$$

$$= 3 + 4 \ln 2 = 3 + \ln 16$$

INTEGRAL #2

**READY,
GET SET,...**

2:00

INTEGRAL #2

$$\int_1^e \frac{1}{x + x \ln x} dx$$

INTEGRAL #2

$$\int_1^e \frac{1}{x + x \ln x} dx$$

$$= \int_1^e \frac{1}{x(1 + \ln x)} dx$$

$$= \int_1^2 \frac{1}{u} du \quad \left[u = 1 + \ln x, \quad du = \frac{1}{x} dx \right]$$

$$= \left[\ln u \right]_1^2 = \boxed{\ln 2}$$

INTEGRAL #3

**READY,
GET SET,...**

2:00

INTEGRAL #3

$$\int_0^{\pi/2} \sin^2 x \cos^3 x \, dx$$

INTEGRAL #3

$$\int_0^{\pi/2} \sin^2 x \cos^3 x \, dx$$

$$= \int_0^{\pi/2} \sin^2 x (1 - \sin^2 x) \cos x \, dx$$

$$= \int_0^1 u^2 (1 - u^2) \, du \quad \left[u = \sin x, \quad du = \cos x \, dx \right]$$

$$= \int_0^1 (u^2 - u^4) \, du = \left[\frac{u^3}{3} - \frac{u^5}{5} \right]_0^1 = \boxed{\frac{2}{15}}$$

INTEGRAL #4

**READY,
GET SET,...**

2:00

INTEGRAL #4

$$\int_0^{\sqrt{\pi}} x \sin^2(x^2) dx$$

INTEGRAL #4

$$\int_0^{\sqrt{\pi}} x \sin^2(x^2) dx$$

$$= \frac{1}{2} \int_0^{\pi} \sin^2 u du \quad [u = x^2, \quad du = 2x dx]$$

$$= \frac{1}{2} \int_0^{\pi} \frac{1 - \cos 2u}{2} du$$

$$= \frac{1}{4} \left[u - \frac{\sin 2u}{2} \right]_0^{\pi} = \boxed{\frac{\pi}{4}}$$

INTEGRAL #5

**READY,
GET SET,...**

2:00

INTEGRAL #5

$$\int_1^e x \ln x \, dx$$

INTEGRAL #5

$$\int_1^e x \ln x \, dx$$

$$\left[\text{integrate by parts: } \begin{array}{l} u = \ln x \quad dv = x \, dx \\ du = \frac{1}{x} \, dx \quad v = \frac{x^2}{2} \end{array} \right]$$

$$= \left[\frac{x^2 \ln x}{2} \right]_1^e - \frac{1}{2} \int_1^e x \, dx$$

$$= \left[\frac{x^2 \ln x}{2} - \frac{x^2}{4} \right]_1^e = \boxed{\frac{e^2 + 1}{4}}$$

INTEGRAL #6

**READY,
GET SET,...**

2:00

INTEGRAL #6

$$\int_9^{64} \sqrt{\frac{1 + \sqrt{x}}{x}} dx$$

INTEGRAL #6

$$\int_9^{64} \sqrt{\frac{1 + \sqrt{x}}{x}} dx$$

$$= \int_9^{64} \frac{\sqrt{1 + \sqrt{x}}}{\sqrt{x}} dx$$

$$= 2 \int_4^9 \sqrt{u} du \quad \left[u = 1 + \sqrt{x}, \quad du = \frac{1}{2\sqrt{x}} dx \right]$$

$$= \left[\frac{4u^{3/2}}{3} \right]_4^9 = \frac{76}{3}$$

INTEGRAL #7

**READY,
GET SET,...**

2:00

INTEGRAL #7

$$\int_1^{2^6} \left(\frac{1}{\sqrt[3]{x^2}} + \frac{1}{\sqrt[6]{x^5}} \right) dx$$

INTEGRAL #7

$$\int_1^{26} \left(\frac{1}{\sqrt[3]{x^2}} + \frac{1}{\sqrt[6]{x^5}} \right) dx$$

$$= \int_1^{26} \left(x^{-2/3} + x^{-5/6} \right) dx$$

$$= \left[3x^{1/3} + 6x^{1/6} \right]_1^{26}$$

$$= 15$$

INTEGRAL #8

**READY,
GET SET,...**

2:00

INTEGRAL #8

$$\int_0^{\pi/3} \sin x \tan^2 x \, dx$$

INTEGRAL #8

$$\int_0^{\pi/3} \sin x \tan^2 x \, dx$$

$$= \int_0^{\pi/3} \sin x \cdot \frac{\sin^2 x}{\cos^2 x} \, dx = \int_0^{\pi/3} \sin x \cdot \frac{1 - \cos^2 x}{\cos^2 x} \, dx$$

$$= - \int_1^{1/2} \frac{1 - u^2}{u^2} \, du \quad \left[u = \cos x, \quad du = -\sin x \, dx \right]$$

$$= \left[\frac{1}{u} + u \right]_1^{1/2} = \boxed{\frac{1}{2}}$$

INTEGRAL #9

**READY,
GET SET,...**

2:00

INTEGRAL #9

$$\int_0^1 \left((x^2 + 10)^2 + (x^2 - 10)^2 \right) dx$$

INTEGRAL #9

$$\int_0^1 \left((x^2 + 10)^2 + (x^2 - 10)^2 \right) dx$$

$$= \int_0^1 (2x^4 + 200) dx$$

$$= \left[\frac{2x^5}{5} + 200x \right]_0^1$$

$$= \frac{1002}{5}$$

INTEGRAL #10

**READY,
GET SET,...**

2:00

INTEGRAL #10

$$\int_0^{\ln \sqrt{3}} \frac{\arctan(e^x)}{e^x + e^{-x}} dx$$

INTEGRAL #10

$$\begin{aligned} & \int_0^{\ln \sqrt{3}} \frac{\arctan(e^x)}{e^x + e^{-x}} dx \\ &= \int_0^{\ln \sqrt{3}} \frac{\arctan(e^x)}{e^x + e^{-x}} \cdot \frac{e^x}{e^x} dx = \int_0^{\ln \sqrt{3}} \frac{e^x \arctan(e^x)}{e^{2x} + 1} dx \\ &= \int_1^{\sqrt{3}} \frac{\arctan u}{u^2 + 1} du \quad \left[u = e^x, \quad du = e^x dx \right] \\ &= \left[\frac{(\arctan u)^2}{2} \right]_1^{\sqrt{3}} = \boxed{\frac{7\pi^2}{288}} \end{aligned}$$

INTEGRAL #11

**READY,
GET SET,...**

2:00

INTEGRAL #11

$$\int_0^1 \frac{1}{\sqrt[3]{19x+8}} dx$$

INTEGRAL #11

$$\int_0^1 \frac{1}{\sqrt[3]{19x+8}} dx$$

$$= \frac{1}{19} \int_8^{27} u^{-1/3} du \quad [u = 19x + 8, \quad du = 19 dx]$$

$$= \frac{1}{19} \left[\frac{3u^{2/3}}{2} \right]_8^{27}$$

$$= \frac{15}{38}$$

INTEGRAL #12

**READY,
GET SET,...**

2:00

INTEGRAL #12

$$\int_0^{\sqrt{\pi}} x^3 \sin(x^2) dx$$

INTEGRAL #12

$$\int_0^{\sqrt{\pi}} x^3 \sin(x^2) dx$$

$$\left[\begin{array}{l} \text{by parts: } u = x^2 \quad dv = x \sin(x^2) dx \\ \quad \quad \quad du = 2x dx \quad v = -\frac{1}{2} \cos(x^2) \end{array} \right]$$

$$= \left[-\frac{x^2 \cos(x^2)}{2} \right]_0^{\sqrt{\pi}} + \int_0^{\sqrt{\pi}} x \cos(x^2) dx$$

$$= \left[-\frac{x^2 \cos(x^2)}{2} + \frac{\sin(x^2)}{2} \right]_0^{\sqrt{\pi}} = \boxed{\frac{\pi}{2}}$$

INTEGRAL #13

**READY,
GET SET,...**

2:00

INTEGRAL #13

$$\int_1^4 \frac{1}{\sqrt{x} e^{\sqrt{x}}} dx$$

INTEGRAL #13

$$\int_1^4 \frac{1}{\sqrt{x} e^{\sqrt{x}}} dx$$

$$= 2 \int_1^2 \frac{1}{e^u} du \quad \left[u = \sqrt{x}, \quad du = \frac{1}{2\sqrt{x}} dx \right]$$

$$= 2 \int_1^2 e^{-u} du = \left[-2e^{-u} \right]_1^2$$

$$= \frac{2}{e} - \frac{2}{e^2} = \frac{2e - 2}{e^2}$$

INTEGRAL #14

**READY,
GET SET,...**

2:00

INTEGRAL #14

$$\int_0^2 \frac{(x-2)^{14}}{(x+2)^{16}} dx$$

INTEGRAL #14

$$\int_0^2 \frac{(x-2)^{14}}{(x+2)^{16}} dx$$

$$= \int_0^2 \left(\frac{x-2}{x+2} \right)^{14} \cdot \frac{1}{(x+2)^2} dx$$

$$= \frac{1}{4} \int_{-1}^0 u^{14} du \quad \left[u = \frac{x-2}{x+2}, \quad du = \frac{4}{(x+2)^2} dx \right]$$

$$= \left[\frac{u^{15}}{15} \right]_{-1}^0 = \frac{1}{15}$$

INTEGRAL #15

**READY,
GET SET,...**

2:00

INTEGRAL #15

$$\int_2^7 \frac{x^{2015} + x^{2014}}{x^{2016} - x^{2014}} dx$$

INTEGRAL #15

$$\begin{aligned} & \int_2^7 \frac{x^{2015} + x^{2014}}{x^{2016} - x^{2014}} dx \\ &= \int_2^7 \frac{x^{2014}(x + 1)}{x^{2014}(x^2 - 1)} dx \\ &= \int_2^7 \frac{x^{2014}(x + 1)}{x^{2014}(x + 1)(x - 1)} dx \\ &= \int_2^7 \frac{1}{x - 1} dx = \left[\ln(x - 1) \right]_2^7 = \boxed{\ln 6} \end{aligned}$$

THANKS FOR PLAYING

LET'S EAT!

(YOU HAVE TWO MINUTES TO FINISH YOUR FOOD)

THE FINAL ROUND BEGINS AFTER DINNER